

A Search for Fine Structure of the Knee in EAS Size Spectra

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28 size spectra of extensive air showers in the knee region from 7 different experiment are analysed consistently. They are fitted by adjusting either 4 or 5 parameters: knee position, power law exponents above and below the knee region, overall intensity and, in addition, a parameter describing the smoothness of the bend. The residuals are then normalized to the same knee position and averaged. When 5 parameters are employed no systematic deviation from a simple smooth knee is apparent at the 1 % level up to about a factor of 4 above the knee. At larger shower sizes a moderately significant deviation can be seen whose shape and position are compatible with a second knee caused by iron group nuclei.

Key words: cosmic rays; knee; EAS

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1 Introduction

The existence of the 'knee' in the spectra of extensive air showers (EASs) has been known by now for more than 40 years [1]. First seen in the number of electrons (the shower 'size') observed near sea level it was later also observed in the muon number [2,3], hadron properties [4,5] and muon densities [6]. In fact it seems to show up in all shower observables if investigated in sufficient detail. Nevertheless its origin is still obscure. Of the explanations proposed two seem to have found more general acceptance.

The first of these relates the knee to the influence of interstellar magnetic fields during propagation of the cosmic ray particles in or leakage from the

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Galaxy or in the course of acceleration for which magnetic fields probably play a major role. Since the radius of curvature of an extremely relativistic particle in a magnetic field is proportionate to its energy E and inversely proportionate to its nuclear charge Z one would expect the knee then to show up at different energies for different nuclear species among the primary particles. In fact one would expect the energy spectra of each element to show a knee at an energy displaced by a factor of Z with respect to that of protons. This was probably first realized by Peters [7] who discussed the implications on various shower observables in great detail². The effect of this on the total energy distribution of cosmic ray particles is illustrated in fig. 1 which shows the result of a simple model calculation based on data compiled by Wiebel-Sooth et al. [8]. These authors give, for each element up to nickel ($Z = 28$), the differential flux at 1 TeV and the exponent of the power law spectrum. These partial spectra were extrapolated up to a knee which was assumed to lie at $\lg(E_{K,Z}[TeV]) = 3.4 + \lg Z$. This corresponds to a knee energy of c. 2.5 PeV for protons. It was then assumed that the exponent increased abruptly at the knee by 0.35 for all elements. All statistical errors (which are of course substantial owing to the extrapolation by some orders of magnitude) were neglected. The full line in fig. 1 represents the sum of all contributions from hydrogen to nickel. The two dotted lines are the extrapolations of the all particle spectrum from below the proton and above the iron knee. As fig. 1 shows the change of slope is, unsurprisingly, not confined to one position or small range along the line. At least three bends are obvious corresponding to hydrogen, helium and the iron group. Hence in this case one would clearly expect a more complicated structure of the spectrum than a simple bend for all shower observables depending on primary energy.

The second proposal, less popular than the first one though recently reasserted by Nikolsky and Romachin [9], attributes the origin of the knee to the properties of high energy interactions in the atmosphere. A change of the spectrum of observables on ground level might of course occur if strong interaction changes by some kind of threshold phenomenon. The knee energy is estimated to be near a few PeV which is approximately a factor of 2 above the highest centre of mass energy available in the laboratory today. Therefore such an effect cannot at present be excluded. Since an EAS induced by a nucleus of mass number A and energy E may, to a reasonable first approximation, be considered as a superposition of A showers induced by nucleons of energy E/A (superposition principle) one would expect, under this assumption, a similar shift of the knee position as described above but by a factor of A instead of Z . Again a more complex structure of the knee would appear natural and a picture very similar to the one shown in fig. 1 would be obtained.

² I thank P. Grieder (Bern) for drawing my attention to this important old reference.

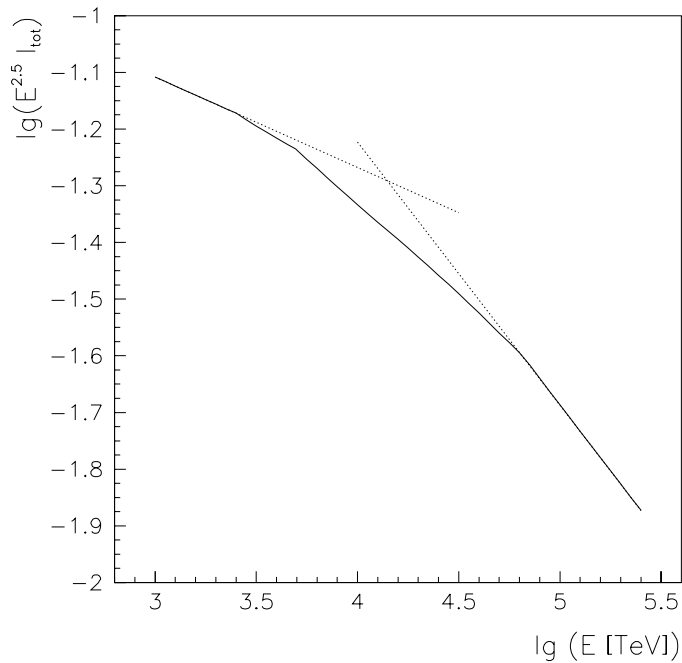


Fig. 1. Theoretical all particle energy spectrum of cosmic rays obtained by extrapolation from lower energies and assuming knee energies increasing in proportion to the nuclear charge. The vertical scale is in arbitrary units.

It should be mentioned that the KASCADE collaboration has recently presented evidence that the knee is to be attributed to light nuclei and that the spectrum of heavier nuclei does not exhibit a change of slope in the vicinity of the 'main' knee. This claim is based on a comparison of electron and muon numbers of EASs with Monte Carlo simulations [10] but also on a phenomenological classification of EAS by their electron to muon ratios [6].

Erlykin and Wolfendale have recently, in a series of papers [11], claimed observational evidence for a more complicated structure of the knee region. This structure which according to the authors does not show up clearly in single measurements but becomes visible when averaging several ones, is attributed to the influence of a recent nearby supernova. It is probably not unfair to say that not many have been convinced by the empirical evidence claimed by the authors. But the underlying idea appears very intriguing and reasonable. The solar system has, during the 4.5 billion years of its existence, probably been passed by several if not many shock fronts from supernovae exploding in its vicinity. One of these clearly must be the most recent one and it appears well conceivable that the cosmic ray spectrum which we observe today is influenced

by this individual event (and hence not typical for the whole Galaxy). The possibility of a single source having a large impact on the energy and mass distributions of cosmic rays at the earth was already realized by Peters [7].

For all these reasons it appears interesting to study the shape of the knee region in more detail and to look as to whether it can be really described by a simple bend between two power laws or whether any of the effects mentioned above can be identified. Except for the work by Erlykin and Wolfendale the author is not aware of any other attempt in this direction. A definite negative result would not only worry some authors but also present difficulties for the usual models of the origin of the knee whose existence, on the other hand, is beyond any doubt.

In this paper I attempt to compare 28 different measured spectra of the electron number N in the knee region. (I drop the usual subscript e because there can be no confusion in this paper.) The data originate from 7 experiments and cover a range of atmospheric depths between 730 and 1250 [g/cm^2]. The electron number (or shower 'size') is probably the shower observable for which the largest amount of measurements exist. The basic procedure adopted is the following: Each spectrum is first fitted separately by an adequate function adjusting either 4 or 5 parameters. In a next step the residual spectra are shifted to the same knee position and averaged. It may be expected that this averaging reduces not only the statistical fluctuations of the measurements but also (at least part of) the systematic ones and hence should make any deviations from the pre-chosen fit function more conspicuous.

The data used and their analysis are described in the following section. In section 3 we compare the results of the analysis with a simple model and present our conclusions in section 4.

2 Data and analysis

2.1 The data base

A list of the spectra analysed in this paper and their sources can be found in table 1. The EAS-TOP experiment is the only one which has published [12]

the N spectra in numerical form³. The CASA data were read from table 6.2 of reference [14] (which is equivalent to fig. of ref. [15]). Such a procedure is of course of limited accuracy and does not exhaust the statistical precision of the data (especially at low shower sizes). All other data sets were made available in numerical form by the authors⁴. Table 1 gives some details of the data together with the quality of the two kinds of fits described in the next subsections. Several experiments registered events in different ranges of zenith distance which then of course correspond to different atmospheric depths. The total data set comprised 784 points which is to be compared with a total of 112 or 140 derived parameters for the four or five parameter fits, respectively.

2.2 Four parameter fits

All fits performed were least squares fits with $\lg N$ and $\lg(N^{2.5}I)$ as the variables and employing the usual weights derived from the quoted errors. Here I is the differential flux. In a first step, the fit function chosen consisted of two power laws connecting continuously at the knee position:

$$I(N) = I_K \left(\frac{N_K}{N} \right)^\gamma \quad (1)$$

Here I_K is the differential flux at the knee position and $\gamma = \gamma_1$ for $N < N_K$ and $\gamma = \gamma_2$ for $N > N_K$.

The quantities I_K , N_K , γ_1 and γ_2 were adjusted to describe the data. After the fit, the differences between observed and fit values were calculated for each of the 28 spectra. These spectra of residuals were then shifted along the $\lg N$ axis to the same knee position. This amounts to choosing $\lg(N/N_K)$ as the new independent variable where N_K is the knee position found for the respective spectrum. The scatter of the data points after these procedures is illustrated in fig. 2 which clearly exhibits the statistical nature of the residuals (and the increase of the errors with increasing N). No errors bars have been drawn because this would completely confuse the picture but most of them are compatible with 0.

As the next step, all data points within horizontal intervals of width 0.1 were averaged neglecting the horizontal uncertainties resulting from the errors of

³ It should be mentioned that the scale factor quoted in the caption of the relevant table 1 of ref. [12] should read 10^{-7} instead of 10^{-8} [13].

⁴ I thank A. Chilingarian (Yerevan), R. Glasstetter (Karlsruhe), G. Heinzelmann (Hamburg), N. N. Kalmykov (Moscow) and M. Nagano (Tokyo) for their invaluable support.

Experiment	atmospheric	number of data points	quality of fit				ref.
	depth [g/cm ²]		4 parameters		5 parameters		
			χ^2	χ^2/F	χ^2	χ^2/F	
AKENO	928	41	52.41	1.416	52.41	1.456	[16,17]
	1021	40	112.87	3.135	112.87	3.225	
	1114	36	39.33	1.229	39.33	1.269	
	1206	34	51.41	1.714	51.41	1.773	
CASA	883	20	37.80	2.362	15.53	1.017	[14]
	911	20	35.63	2.227	19.41	1.137	
	941	20	58.02	3.627	43.29	1.699	
	972	20	34.63	2.164	29.40	1.400	
	1006	20	29.65	1.853	18.87	1.122	
	1042	20	47.84	2.990	32.73	1.477	
	1081	20	29.56	1.847	28.85	1.387	
EAS-TOP	835	30	26.51	1.020	26.51	1.061	[12]
	880	30	17.82	0.685	13.98	0.559	
	920	30	13.42	0.516	13.12	0.525	
	960	30	20.34	0.782	16.77	0.671	
	1000	30	6.14	0.236	6.14	0.246	
	1040	30	9.91	0.381	7.96	0.318	
HEGRA	820	12	16.76	2.095	16.68	2.383	[18]
KASCADE	1047	33	87.62	3.021	42.80	1.529	[3,19]
	1098	34	42.39	1.413	26.51	0.914	
	1149	34	39.11	1.304	25.57	0.882	
	1200	33	54.80	1.890	24.97	0.892	
	1251	30	31.64	1.217	25.53	1.021	
MAKET-ANI	731	30	58.51	2.250	52.84	2.114	[20]
	804	28	19.74	0.822	19.69	0.856	
	876	27	27.00	1.174	26.93	1.224	
	949	27	31.28	1.360	28.41	1.292	
MSU	1068	25	85.31	4.062	84.84	4.142	[21]

Table 1

Data sets used in this analysis and fit results. The number of degrees of freedom F is the number of data points minus 4 or 5, respectively.

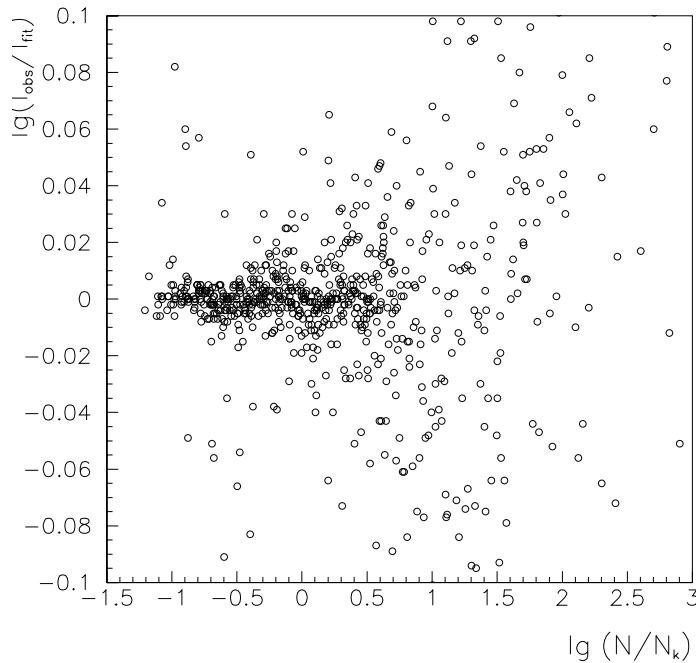


Fig. 2. Differences between observed and fit values of the differential flux normalized to the same knee position. The data are based on fits with four parameters. Each symbol represents one data point from one of the 28 spectra. No error bars have been drawn in order not to confuse the picture but most are compatible with 0.

N_K . The latter were smaller than the chosen bin width for almost all spectra. The number of data points within a given interval was above 30 in the vicinity of the knee and dropped to near 1 at the extreme ends of the total range because the N ranges of the various measurements did not coincide. It should be mentioned that most of the original data were also binned in intervals of 0.1 width so this choice was very natural.

The averaging procedure described actually amounts to taking the geometric means of I_{obs}/I_{fit} (with adequate weights). This eliminates all sensitivity to the absolute normalization of the data. There is reason to suspect that the quantity which different experiments call shower size N is not the same. This is most obvious when the influence of muons on the experimental results is considered. The electron detectors, in all experiments scintillation counters, are also sensitive to muons. Some experiments, such as KASCADE [22,23], put a lot of effort into correcting for this effect. This is impossible, on the other hand, if muons are not measured independently as is the case in some of the experiments included in this analysis. Also different detector thresholds or saturation effects may lead to systematic differences. The advantage of choosing $lg(N/N_K)$ as a new variable is that constant scale factors cancel. Although

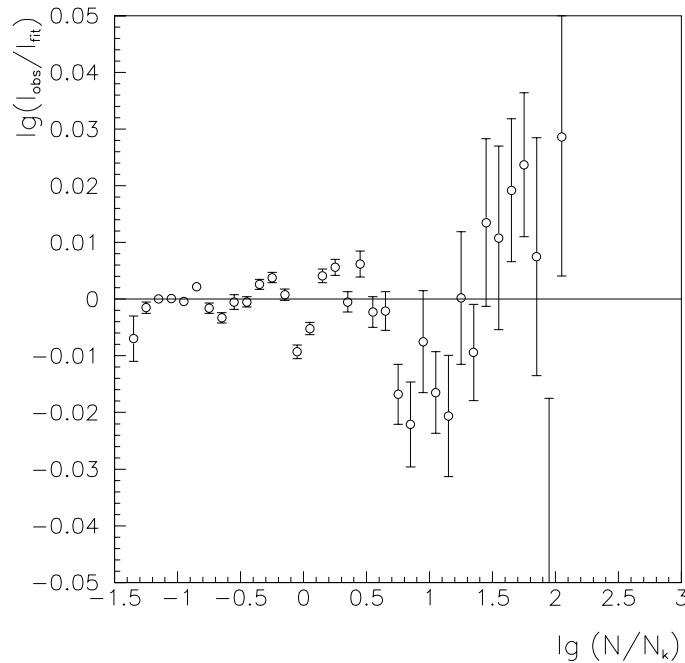


Fig. 3. Averaged differences between observed and fit values as shown in fig. 2 for the fits with 4 parameters.

not all of the effects mentioned above will lead to modifications of N by a constant factor at least some instrumental effects will be reduced or even removed.

The weighted averages thus obtained are displayed in fig. 3 with their statistical errors (which have been multiplied, as usual, by the root of χ^2/F if the latter was larger than 1). A number of points at the upper end of the spectrum have been omitted here because of their huge errors. These were all compatible with 0 within their statistical errors. In spite of the considerable errors the mean residuals can hardly be said to scatter statistically. The most significant deviations, at the two points neighbouring the knee position, amount to 7.6σ and 4.6σ and are therefore highly significant.

A large discrepancy in the immediate vicinity of the knee position is in no way surprising because several experiments have shown that the slope of the spectrum does not change abruptly at the knee but rather smoothly over a finite range of lgN . The sign of the deviation is precisely as would be expected in such a situation. So the most prominent deviation just reflects a well known and expected feature. On the other hand, this result also shows that the procedure adopted in this paper is suitable to reveal more clearly a structure where it is present. But obviously a more suitable fit function has to be employed.

It is worthwhile mentioning that the knee positions resulting from the fits show the well known dependence on atmospheric depth. Five of the seven experiments with 18 spectra are in good agreement with a simple linear dependence. The other two can be made to agree when the shower sizes are scaled down by 30% and 70%. The latter would not affect the present analysis in any way, of course. The scatter of the other parameters is clearly larger than the quoted statistical errors would indicate. This is subject of further analysis. Since in this study we are focussing on deviations from a smooth size dependence in a limited size range I would not expect such (apparently systematic) differences between experiments to influence my conclusions.

2.3 *Five parameter fits*

Several different methods have been used to describe a smooth knee all of which seem to work reasonably well. A very popular procedure is to select a finite interval in the vicinity of the knee and then first fit power laws to the regions above and below this interval. In a second step these functions are then connected smoothly, e.g. by a polynomial. A disadvantage of this method is, in my opinion, that at least two but usually more additional parameters are introduced (which does not seem to be always realized).

Ter-Antonyan and Haroyan [24] have used a function which avoids this disadvantage. Their expression

$$I(N) = I_K \cdot \left(\frac{N}{N_K}\right)^{-\gamma_1} \cdot \left(1 + \left(\frac{N}{N_K}\right)^\epsilon\right)^{(\gamma_1 - \gamma_2)/\epsilon}$$

can be shown to approach pure power laws far from the knee and to tend to eq. (1) for $\epsilon \rightarrow 0$. The additional parameter ϵ does not lend itself to a direct physical interpretation, though. Also the two regions above and below the knee are not described in a formally equivalent way.

I have adopted a different approach. It consists of folding eq. (1) with a normalized distribution of finite variance. This implies a model in which a hypothetical underlying spectrum with a sharp knee is observed with finite resolution. The exact shape of the 'resolution function' is not really important but if a

log-normal distribution is chosen the following closed expression is obtained:

$$I(N) = I_K \left(e^{\sigma^2 \gamma_1^2 / 2} \Phi(u_1) \left(\frac{N_K}{N} \right)^{\gamma_1} + e^{\sigma^2 \gamma_2^2 / 2} [1 - \Phi(u_2)] \left(\frac{N_K}{N} \right)^{\gamma_2} \right) \quad (2)$$

$$u_i = \sigma \gamma_i - \frac{\ln(N/N_K)}{\sigma}$$

Here σ is the standard deviation of the log-normal distribution and $\Phi(u)$ is the error integral:

$$\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^u e^{-x^2/2} dx$$

It is again straightforward to show that eq. (2) approaches power laws at sufficient distances from the knee and tends to eq. (1) for $\sigma \rightarrow 0$. As far as the physical interpretation is concerned one should realize that σ does not primarily represent instrumental effects. It is well known from simulations of EAS development in the atmosphere that the shower size N fluctuates considerably for events of fixed primary energy, mass and direction of incidence. The effects of these fluctuations are also incorporated in the parameter σ .

I should like to point out that it is, in my opinion, very much a matter of taste which of the described approaches is chosen. Since all describe the experimental spectra well within observational uncertainties they will all lead to the same conclusions.

The fitting and averaging was then performed as described in the preceding subsection. Fig 4 displays the result. The residuals are now below 0.005 (which corresponds to a difference between fit and measurement of less than $\simeq 1\%$) for the whole lower part of the spectrum, up to $\lg(N/N_K) \simeq 0.6$.

The remaining deviations in the upper part amount to $\simeq 10\%$ on the positive side and $\simeq 5\%$ on the negative side. Their regular pattern with increasing size can hardly be called statistical in spite of the fact that only two of the last 15 data points differ from 0 by more than 2 standard deviations. A possible explanation is discussed in the next section where the origin of the dotted curve in fig. 4 will also be explained.

One might argue that several data points in the lower size range clearly differ from 0 by a similar amount statistically or even recognise an oscillatory

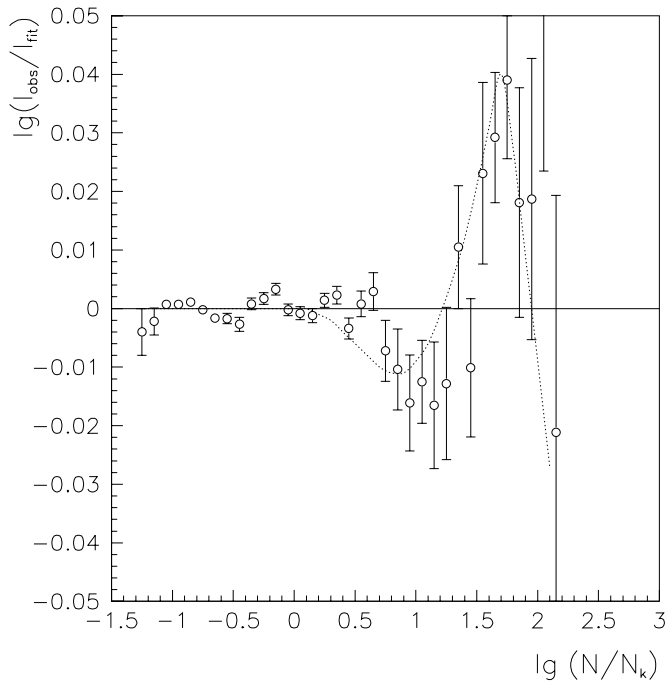


Fig. 4. Averaged differences between observed and fit values based on fits with 5 parameters. The dotted line is discussed in section 3.

structure. But this deviation is on the 1 % level and hence a factor of 5 to 10 smaller than the one in the upper size range. I would tend to argue, to the contrary, that an agreement between various experiments on this level is rather impressive and would hesitate to draw far reaching conclusions from these observations.

3 Discussion

The results displayed in fig. 4 show clearly that the lower part of the observed size spectrum is well described by two power laws and a simple knee. I find it difficult to believe, on the other hand, that the discrepancy visible in the upper part of the size range is purely statistical, in spite of the large errors of the individual points (but this is of course left to the reader's own judgement). It is much more difficult to assess possible systematic errors. I will come back to the latter question later in this section and first turn to a possible explanation of the observed deviations assuming they are real.

Let us suppose that the real size spectrum exhibits two knees displaced by one

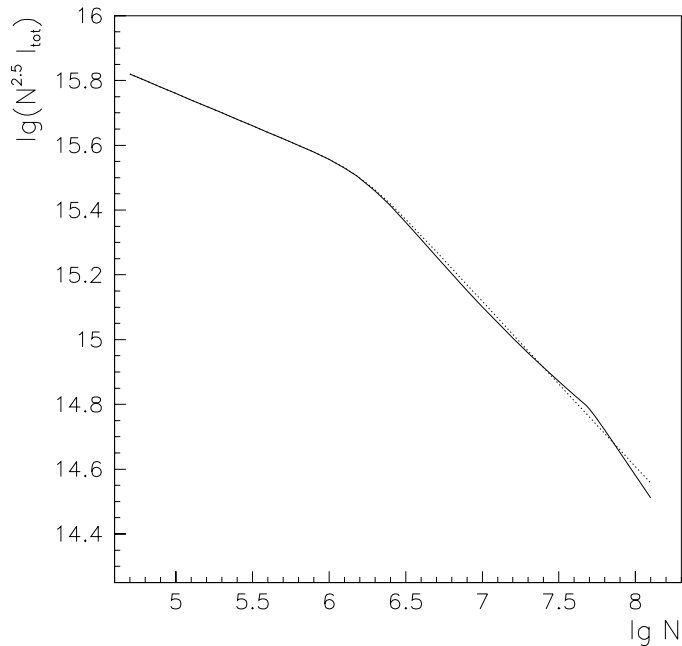


Fig. 5. Calculated size spectrum consisting of two functions of the type shown by eq. (2) but displaced horizontally (full line). The dotted curve is of the same type and its parameters have been adjusted to give an approximate description of the full line. The vertical scale is in arbitrary units.

or two orders of magnitude with respect to each other. This is not unreasonable in view of the considerations extended in the introduction and illustrated in fig. 1. Then a situation might occur which is sketched in fig. 5. Here the full line represents a hypothetical spectrum with two knees. If one tries to fit this spectrum with an expression allowing only one knee this will be adjusted to the lower one because it is more pronounced. Also statistical errors of the data points increase with increasing shower size and hence the lower points carry larger weights. But the deviation of the data from a pure power law in the higher size range will influence the fit and might lead to a situation illustrated by the dotted line in fig. 5. This would then result in differences between data and fit which agree qualitatively with those observed in fig. 4. I admit that the separation of the full and dotted lines in fig. 5 does not look dramatic but one should be aware of the difference in the vertical scales between figs. 4 and 5.

It is possible to extend this conjecture a step further. For this let us try to estimate the *size* spectrum resulting from an *energy* spectrum as displayed in fig. 1. Essentially all simulations of EASs lead to a power law relation between shower size and energy for primary protons, at least in the range of energy

and atmospheric depth relevant for this analysis:

$$N_p = a E_p^\delta$$

The corresponding relation for a heavier primary may then be estimated by help of the superposition principle invoked already in the introduction:

$$N_A = A a \left(\frac{E_A}{A} \right)^\delta$$

Here E_A denotes the energy of a nucleus of mass number A . Assuming again, as in the introduction, that the knee position in energy is shifted by a factor of Z , $E_{K,A} = Z E_{K,p}$, one obtains for the shift of the knee in shower size

$$N_{K,A} = A a \left(\frac{E_{K,A}}{A} \right)^\delta = a Z^\delta A^{1-\delta} E_{K,p}^\delta$$

$$\lg N_{K,A} = \lg N_{K,p} + \delta \lg Z - (\delta - 1) \lg A$$

So in this simple model the shift depends only on mass and charge of the nuclei. The second column of table 2 shows these shifts for some abundant elements. The value of $\delta \simeq 1.30$ which is the only additional information needed, was calculated from simulation results reported in tables 20 and 26 of [25] (for the QGS-JET model; the other 4 models given in the same tables differ from this by at most 0.04). The separation between hydrogen and helium is reduced as compared to the corresponding shift in the energy spectrum due to the difference in mass to charge ratio. So the knees of these elements would merge if the smoothness is taken into account. Hence in this case the knee observed in the size spectrum has probably to be attributed to the combined effect of protons and helium. It is obvious that the shift of the knee for iron is of the right order of magnitude to explain a deviation as seen in fig. 4.

If the other (less popular) proposal for the origin of the knee is adopted, $E_{K,A} = A E_{K,p}$, one obtains in a similar way

$$\lg N_{K,A} = \lg N_{K,p} + \lg A$$

The corresponding values have also been entered into table 2. Intriguingly, the position of the iron knee obtained under the assumption of $E_{K,A} \propto A$ is in better quantitative agreement with the deviations observed in fig. 4. On the other hand, the knees originating from protons and helium should then appear well separated of which there is not much evidence in the figure.

Element	$lg(N_{K,A}/N_{K,p})$	
	$E_K \propto Z$	$E_K \propto A$
He	0.21	0.60
C	0.69	1.08
O	0.81	1.20
Ne	0.91	1.30
Mg	0.99	1.38
Si	1.06	1.45
Fe	1.32	1.75

Table 2

Estimated shifts of the knee in the size spectrum with respect to the proton knee under the simple assumptions that the knee energy is either proportionate to charge or to mass of the primary nuclei.

In order to check this possibility further I have calculated the following 'toy' model. I have assumed that the size spectrum can be described by the sum of two functions of the type given in eq. (2) with different intensities, the same slopes and the two knees separated by $\Delta lg N_K = 1.7$. This model spectrum then has two free parameters to adjust, the ratio of intensities and the change of the power law exponents. For the latter $\Delta\gamma = 0.5$ was assumed. The parameters σ were chosen to reproduce the variance of the simulation results in the quoted tables of ref. [25] and were 0.44 for protons and 0.08 for iron. This choice neglects all instrumental effects on the size resolution and should therefore represent a lower limit. This calculated spectrum was then fitted by a single function of the same type with the knee at the same position as the lower of the other two. Intensity and slope below the knee were taken to be the same as those of the model spectrum, and the slope above the knee adjusted. The difference of the model and fit spectra are shown in fig. 4 as the dotted line. It should be emphasized that the width of the knees were not varied but kept fixed at the values quoted above. The three new parameters (relative intensity, change of the exponent of the model function and slope above the knee of the fit function) were adjusted to some extent but no serious attempt was made to obtain a perfect fit. For this all elements expected to be present in primary cosmic rays should have to be taken into account. Also there is no compelling reason to believe that all partial spectra have the same exponents. This would then leave more parameters to adjust than data points in fig. 4. Although the dotted line does not reproduce the data perfectly the model gives a reasonable description in view of its crudeness. Hence the data can be said to be in agreement with a second component in the overall spectrum exhibiting a knee at higher shower size. It remains surprising, though, that the displacement is clearly in better agreement with a proportionality of the knee position to mass rather than to charge of the primary nuclei (if the sec-

ond component is attributed to iron which appears to be the most reasonable choice).

This is probably the right moment to stop speculating and to turn to the thornier question of systematic errors. It should be mentioned that the number of experimental points averaged drops from 29 at $lgN = 0.6$ to 7 at 2.1. So the region where the deviation is observed covers the highest data points of several experiments and these might be under suspicion to suffer from saturation effects. Saturation will lead to an overestimate of the differential flux increasing towards the end of the range of measurement. Therefore it is, in my opinion, not possible to exclude at this moment systematic effects as a possible origin of the observed deviations without more detailed investigations. These require a better knowledge of experimental details and have to be performed for each experiment separately. This is beyond the scope of the present paper.

4 Summary

I have presented an analysis of 28 size spectra from 7 different experiments. Their results can be well represented by two power laws with a smooth knee in between up to a shower size a factor of $\simeq 4$ above the bend. At larger sizes a deviation occurs which appears statistically sound though not overwhelming. The maximum of this deviation roughly coincides with the position where one would expect a knee originating from primary iron nuclei if the knee in energy is assumed to be shifted in proportion to primary mass. I realize of course that, to the best of my knowledge, no such feature has ever been seen before in *size* spectra although a second knee was seen in the derived *energy* spectrum by the AKENO group [17]. (The deviations claimed by Erlykin and Wolfendale [11] are closer to the main knee position and cannot therefore be identified with the ones observed here.) But on the other hand no analysis so far has taken advantage of the statistics of 28 spectra from 7 experiments. The effect shows up in a region where detectors might start to saturate which may lead to distortions of the spectrum. Hence systematic errors cannot, in my opinion, presently be ruled out as the origin of the observed fine structure.

I would consider it worthwhile to study the data from experiments with high statistics and high detector quality in more detail, especially as far as saturation effects are concerned, in order to verify or not the existence of this fine structure. One way of checking the influence of saturation is to compare spectra taken by the same experiment in different bins of zenith distance. The measured size of a primary of given energy and mass decreases with increasing

zenith angle. This results in a shift of the knee position to lower sizes with increasing zenith distance. This shift has been observed by many experiments and is also clearly visible in the data of the 5 experiments which have contributed more than one spectrum to this investigation. Hence vertical showers should saturate at lower values of $lg(N/N_K)$. If the same structure is observed at all zenith distances the influence of saturation can be ruled out. The range of shower sizes of such a study should of course extend up to at least two orders of magnitude above the 'main' knee. This implies a reduction of particle flux by about 4 orders of magnitude and hence requires very good statistics as well as detectors of a considerable dynamic range.

It should be mentioned that the analysis described in section 2 of course also yields numerical values for the 4 or 5 parameters and their statistical errors and it appears interesting to compare the results from the various experiments and study their dependence on atmospheric depth. This will be the subject of a forthcoming paper.

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References

- [1] G. V. Kulikov and G. B. Khristiansen, Soviet Phys. JETP **35** (1959) 441
- [2] J. N. Stamenov et al., Trudy FIAN SSSR **109** (1972) 132
- [3] R. Glasstetter (KASCADE collaboration), Proc. 25th Internat. Cosmic Ray Conf. Durban 1997, contrib. HE 2.1.5;
- [4] H. Yoshii, J. Phys. Soc. Japan **32** (1972) 295
- [5] J. R. Hörandel et al. (KASCADE collaboration), Proc. 26th Internat. Cosmic Ray Conf., Salt Lake City 1999, contrib. HE 2.2.41

- [6] A. Haungs et al. (KASCADE Collaboration), Proc. 26th Internat. Cosmic Ray Conf., Salt Lake City 1999, contrib. HE 2.2.02; T. Antoni et al. (KASCADE Collaboration), *Muon density measurements with the KASCADE central detector*, Astropart. Phys., in press
- [7] B. Peters, Nuovo Cim. **22** (1961) 800
- [8] B. Wiebel-Sooth, P. L. Biermann and H. Meyer, Astron. Astrophys. **330** (1998) 389
- [9] S. I. Nikolsky and V. A. Romachin, Phys. Atom. Nucl. **63** (2000) 1799
- [10] K. H. Kampert et al. (KASCADE Collaboration), Proc. 26th Internat. Cosmic Ray Conf., Salt Lake City 1999, contrib. OG 1.2.11
- [11] A. D. Erlykin and A. W. Wolfendale, Astropart. Phys. **7** (1997) 1, 203, **10** (1999) 69; J. Phys. G: Nucl. Part. Phys. **23** (1997) 979, **26** (2000) 203; Astron. Astrophys. **350** (1999) L1, **356** (2000) L63
- [12] M. Aglietta et al. (EAS-TOP Collaboration), Astropart. Phys. **10** (1999) 1
- [13] G. Navarra, private communication (2000)
- [14] M. A. K. Glasmacher, PhD Thesis, The University of Michigan (1998)
- [15] M. A. K. Glasmacher et al. (CASA-MIA Collaboration), Astropart. Phys. **10** (1999) 291
- [16] M. Nagano et al., J. Phys. G: Nucl. Part. Phys. **10** (1984) 1295; M. Nagano, private communication (2000)
- [17] M. Nagano et al., J. Phys. G: Nucl. Part. Phys. **18** (1992) 423
- [18] G. Heinzelmann, private communication (2000)
- [19] R. Glasstetter, private communication (2000)
- [20] A. Chilingarian, private communication (2000); A. Chilingarian et al., Proc. 26th Internat. Cosmic Ray Conf., Salt Lake City 1999, contrib. HE 2.2.9
- [21] N. N. Kalmykov, private communication (2000)
- [22] H. O. Klages et al. (KASCADE Collaboration), Nucl. Phys. B (Proc. Suppl.) **52B** (1997) 92
- [23] J. H. Weber, PhD Thesis, University of Karlsruhe (1999); Report FZKA 6339, Forschungszentrum Karlsruhe (1999) (in German)
- [24] S. V. Ter-Antonyan and L. S. Haroyan, hep-ex/0003006
- [25] J. Knapp, D. Heck and G. Schatz, Report FZKA 5828, Forschungszentrum Karlsruhe (1996)